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**TESTS AND COMPARISONS OF
GRAVITY MODELS USING CAMERA
OBSERVATIONS OF GEOS-I AND GEOS-II**

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FEBRUARY 1970



— GODDARD SPACE FLIGHT CENTER —
GREENBELT, MARYLAND

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ABSTRACT

Optical observations of the GEOS satellites are used to obtain orbital solutions with different sets of geopotential coefficients. The solutions are compared before and after modification to high order terms (necessary because of resonance) and then are analyzed by comparing subsequent observations with predicted trajectories. The most important source of error in orbit determination and prediction for the GEOS satellites is the poorly modeled effect of resonance found in most published sets of geopotential coefficients. Modifications to the sets yield greatly improved orbits in most cases.

The sets of coefficients analyzed are APL 3.5, NWL5E-6, Köhnlein (1967), Rapp (1967), Kaula (1967), Smithsonian Astrophysical Observatory (SAO)M-1 (1966), SAO COSPAR (1969) and SAO 1969. The SAO 1969 model generally gives better orbital fits and prediction results than the models quoted above. However even this model can be improved by corrections to resonant coefficients.

The results of these comparisons suggest that with the best optical tracking systems and gravity models, satellite position uncertainty can reach 50-100 meters during a heavily observed 5-6 day orbital arc.

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INTRODUCTION

For GEOS-I and II data to be utilized for tracking system intercomparison, calibration, and station location determination, satellite positions must be accurately determined. Thus, an accurate model of the forces on the satellite is essential. For the GEOS satellites, the small effects of drag and radiation pressure are easily modeled. The much larger effects of the geopotential present a more serious problem. This study consists of analysis, comparison, and modification of existing geopotential models.

The sets of geopotential coefficients (gravity models) were used in the NONAME Cowell-type orbit determination program (Reference 1) to obtain orbital solutions. These were then compared by examining rms of fit, differencing the fitted orbits, and predicting the orbits through later GEOS data.

All except the SAO 1969 solutions were obtained using the SAO C-7 Standard Earth (Reference 2). The gravity models studied were SAO M-1, SAO COSPAR (1969), SAO 1969, APL 3.5, NWL5E-6, Kaula 1966, Köhnlein 1967, and Rapp (1967) (References 3-10).

The most significant defect of most published gravity models, particularly the older ones, for precision orbit determination is their lack of high-order terms to model the shallow resonances which exist for all satellites. For GEOS-I and II, perturbations due to resonant effects amount to approximately 500 meters along track. In most cases, improvements could be obtained by modifying the resonant coefficients of a model.

It is a common practice in orbit determination to represent an orbit by more than 6 parameters in order to absorb model errors. Since we were trying to discover model errors we solved only for the minimum set of 6 elements.

SECTION 1

ORBITAL CHARACTERISTICS OF GEOS-I AND II

GEOS-I and II are nearly ideal for gravity model testing because of the extent of coverage by accurate tracking instruments and low drag and radiation pressure effects. The orbital inclinations of GEOS-I and II differ markedly, thus ensuring that conclusions from tracking results of both satellites have some generality.

1.1 ORBITAL SPECIFICATIONS

Table 1 presents the orbital specifications for GEOS-I and II. GEOS-I has a period of approximately two hours and an orbital inclination of 59°4; GEOS-II has a somewhat shorter period and a much higher inclination of 105°8. Both orbits are nearly circular with perigees above 1000 km and small area-to-mass ratios. Consequently, drag can be ignored and radiation pressure is small and easily modeled.

Table 1
Orbital Elements of GEOS I and II

	GEOS I	GEOS II
Epoch	January 2, 1966	April 28, 1968
Apogee Height	2273 Kilometers	1569 Kilometers
Perigee Height	1116 Kilometers	1077 Kilometers
Eccentricity	0.07	0.03
Inclination	59.4 Degrees	105.8 Degrees
Anomalistic Period	120.3 Minutes	112.1 Minutes

Table 2 presents the along-track effects of the SAO M1 geopotential up to (6,6) for GEOS-I and II. A few terms of higher degree such as (8,1) also have effects of about 20 m. Notice that GEOS-II is more perturbed by the geopotential than is GEOS-I due to the smaller orbital semimajor axis and higher inclination.

1.2 RESONANCE

GEOS-I is resonant with 12 th order (m) terms of the geopotential and GEOS-II is resonant with terms of the 13th order. The result is a perturbation along track of about 1/2 km in each case.

Table 3 shows the expected along track effects assuming the normalized coefficients follow the rule ($\bar{C}_{n,m}, \bar{S}_{n,m} \approx 10^{-5} / n^2$). Although both satellites have nearly circular orbits, the even degree (n) terms have important effects. These terms contain a factor proportional to the eccentricity and are called eccentric resonant terms. Table 3 indicates the necessity of modeling these terms.

Table 2
Along Track Effects of Low Degree
and Order Terms
Based on the SAO M-1 Coefficients

n,m	METERS	
	GEOS I	GEOS II
2,2	350	500
3,1	80	150
3,2	50	30
3,3	25	50
4,1	130	200
4,2	50	50
4,3	85	75
4,4	10	20
5,1	<10	15
5,2	35	10
5,3	15	15
5,4	10	<10
5,5	<10	10
6,1	<10	10
6,2	35	<10
6,3	<10	<10
6,4	25	<10
6,5	25	30
6,6	<10	<10

Table 3
Along Track Effects of Resonant Terms
Assuming $\bar{C}_{n,m}, \bar{S}_{n,m} \approx 10^{-5} / n^2$

GEOS I		GEOS II	
Beat Period = -7.1 Days		Beat Period = -6.5 Days	
n,m	Meters	n,m	Meters
12,12	~170	13,13	~400
13,12	400	14,13	150
14,12	280	15,13	280
15,12	225	16,13	50
16,12	80	17,13	100
17,12	50	18,13	40
18,12	50	19,13	10
19,12	40	20,13	20
20,12	30	21,13	25
21,12	25	22,13	10
22,12	15	23,13	20
n > 23	<10	n > 24	<10

Both orbits contain a large number of resonant terms significantly affecting the satellite position. However, an extensive set of resonant coefficients may not be absolutely necessary for accurate orbit determination and prediction. Resonant terms of either even or odd degree perturb the orbit with about the same frequency (Kaula Reference 11).

Thus most or all of the effects of all the resonant

terms can be absorbed by solving for one or two pairs of them. The good results obtained in this study would not have been possible otherwise, since the number of resonant terms in most models is relatively small. A detailed analysis of GEOS-II orbital resonance is given in Reference 12.

SECTION 2

DESCRIPTION OF DATA SETS

2.1 DATA TYPES

Three types of camera data were used for this study. These are Baker-Nunn (SAO), PC-1000 (United States Air Force), and MOTS 40" (NASA STADAN and SPECT). The same a priori standard deviations on the measurements were used:* two seconds of arc on all declination measurements and $2/\cos$ (declination) seconds of arc on all right ascension measurements.

It should be noted that although the measurements are assumed to have the same accuracy, the locations of most of the Baker-Nunn stations are possibly better known.

2.2 ORBITAL ARC DESCRIPTIONS

Four approximately 5-1/2 day arcs were chosen for this study; two GEOS-I and two GEOS-II arcs. Table 4 describes these arcs.

Table 4
Description of Orbital Arcs and Data Sets

Satellite	Period	Camera Type	No. OBS
GEOS I	Dec. 31, 1965 - Jan. 5, 1966	MOTS, PC-1000 (10%)	1057
	July 11 - 16, 1966	Baker-Nunn (90%)	1766
GEOS II	Apr. 28-May 4, 1968	MOTS 40" (100%)	1098
	Sept. 13-22, 1968	Baker-Nunn (95%)	1388
		MOTS 40" (5%)	

The July 11-16, 1966, GEOS-I arc is predominantly Baker-Nunn (SAO) data from the original twelve best located SAO tracking stations, while the Dec. 31-Jan. 5, 1966, GEOS-I arc is mostly MOTS 40" data.

The Apr. 28-May 4, 1968 GEOS-II arc also consists mostly of MOTS data. The second GEOS-II arc, Sept. 16-22, 1968 is largely Baker-Nunn data from less accurately located SAO stations and, at six days, is the longest of the four arcs.

Solutions from the best of the GEOS-I arcs, the July arc and best of the GEOS-II arcs, the April-May arc, were used in the prediction results and satellite position comparisons presented in Section 3. Thus it is important to note the following: The Baker-Nunn cameras in the GEOS-I

*Used in making up the covariant weight matrix.

arc are much more widely distributed about the Earth than are the MOTS cameras in the GEOS-II arc. The GEOS-I arc has a significant number of observations from stations in Spain, India, Australia and Hawaii, as well as North and South America. The tracking stations used in the GEOS-II arc are almost totally located in North-Central America, with one station in Chile and two in Africa. Such a difference between the two arcs is more likely to cause differences in prediction results than in orbit determination.

SECTION 3 COMPARISON OF GRAVITY MODELS

3.1 MODEL AND RESONANT TERM DESCRIPTIONS

Table 5 summarizes the geopotential models evaluated. Until the presentation of the SAO 1969 model at the COSPAR meeting in Prague by E. M. Gaposchkin in May, 1969, the 1966 SAO M-1 was the most extensive published model based on satellite data alone. Derived by the Smithsonian Astrophysical Observatory from Baker-Nunn optical observations of 16 satellites, the set is complete to (8,8) with 46 additional coefficients of higher degree totalling 122 coefficients. The two recent SAO models, the COSPAR and the 1969, are complete to (14,14) and (16,16) respectively with many additional coefficients of higher degree and were derived from a combination of optical, Goddard Range and Range Rate and laser data, from 24 satellites. The 1969 model also incorporates gravimetric data.

Table 5
Geopotential Models

SAO M-1 (1966)	Complete to (8,8)	122 Coefficients
SAO COSPAR (1969)	Complete to (14,14)	280 Coefficients
SAO 1969	Complete to (16,16)	314 Coefficients; Includes Gravimetric Data
APL 3.5 (1965)	Complete to (8,8)	84 Coefficients
NWL 5E-6 (1965)	Complete to (7,6)	64 Coefficients
KAULA (1966)	Complete to (7,5)	99 Coefficients
Köhnlein (1967)	Complete to (15,15)	250 Coefficients; Includes Gravimetric Data
Rapp (1967)	Complete to (14,14)	219 Coefficients; Includes Gravimetric Data

The APL 3.5 model was derived from Tranet Doppler satellite observations by the Applied Physics Laboratory. This set is complete to (8,8) with additional higher degree terms totalling 84 coefficients.

The Naval Weapons Laboratory derived the NWL 5E-6 model also using Tranet Doppler data. It is complete to (7,6) with a few additional higher degree coefficients.

The Kaula model, derived in 1966 from a combination of Tranet Doppler and optical observations of 12 satellites is complete to (7,5) with a few higher degree coefficients making a total of 99 coefficients.

The Köhnelein and Rapp models, complete to (15,15) and (14,14) respectively, were derived by combining gravimetric measurements with the SAO M-1 coefficients in 1967.

Table 6 describes the resonant terms used to modify the geopotential models. The Gaposchkin and Veis values for (13,12), (14,12) and (15,12) (Reference 13) are used with the 1966 M-1 values for (12,12) and replace all existing 12th order terms in the modified models in GEOS I arcs. The Yionoulis (Reference 14) values for (13,13), (15,13) and (17,13) replace all existing 13th order terms in the modified models in GEOS II arcs. The Douglas and Marsh values for (14,13) (Reference 12) are used with the Yionoulis 13th order terms and together replace all existing 13th order terms in the modified models in the April-May GEOS II arc.

Table 6
Sets of Resonant Coefficients Used
To Modify Geopotential Models

Source	Resonant coefficients	Comments
Gaposchkin and Veis (1967)	$C_{13,12} = -1.26 \times 10^{-19}$ $S_{13,12} = 1.16 \times 10^{-19}$ $C_{14,12} = 1.40 \times 10^{-21}$ $S_{14,12} = -1.32 \times 10^{-20}$ $C_{15,12} = -1.38 \times 10^{-20}$ $S_{15,12} = -1.9 \times 10^{-21}$	To be used with 1966 M-1 (12,12) where $C_{12,12} = -2.78 \times 10^{-19}$ $S_{12,12} = 7.18 \times 10^{-21}$
Yionoulis (1968)	$C_{13,13} = -2.39 \times 10^{-20}$ $S_{13,13} = 2.12 \times 10^{-21}$ $C_{15,13} = -7.7 \times 10^{-22}$ $S_{15,13} = -3.74 \times 10^{-22}$ $C_{17,13} = 1.59 \times 10^{-23}$ $S_{17,13} = 2.8 \times 10^{-23}$	
Douglas and Marsh (1969)	$C_{14,13} = 5.7 \times 10^{-22}$ $S_{14,13} = 6.5 \times 10^{-21}$	To be used with Yionoulis (1968) for the April-May arc.

For the SAO 1969, the appropriate C6 station positions were used. All other cases used the appropriate C7 positions.

3.2 ORBIT DETERMINATION

The quality of a determined orbit is measured by the root mean square (rms) of the data points about the orbital solution. Tables 7 and 8 present the rms's of fit in seconds of arc about the orbital solutions for each of the models studied, with and without modification of resonant terms.

With the exception of the 1969 SAO models, the unmodified gravity models gave relatively poor fits. Modifications for resonance greatly reduced the rms's of most of the models. The exception, the Kaula model, gave best results for GEOS-I before modification.

As mentioned in Section 1.2 and shown conclusively in Tables 7 and 8, resonance for GEOS-I and II is important. The Gaposchkin and Veis 12th order terms for GEOS I and the Yionoulis 13th order terms for GEOS-II produced significantly better fits in the models which contained an insufficient number of accurate resonant terms.

It is interesting to notice in Table 7 that the original SAO M-1 12th order coefficients produced a very poor fit for the July arc and a very good one for the December-January arc. The reason for this is not yet clear. The July arc residuals indicate that the high rms is due to resonance: a distinct six-day period can be seen on a plot of the residuals.

Table 7
Rms's about Fitted Orbits
GEOS I

ARC 1: July 11-16, 1966
(1766 observations)

Model	Rms (secs of arc)*	
	unmodified	modified**
SAO M-1	19.04	2.52
SAO COSPAR (No 11th)	2.42	- -
SAO 1969	1.93	- -
Köhnlein	14.65	2.89
Rapp	7.81	6.91
NWL 5E-6	11.82	3.33
APL 3.5	13.51	6.64
Kaula	5.80	5.95

ARC 2: December 31 - January 5, 1966
(1057 observations)

Model	Rms (secs of arc)	
	unmodified	modified*
SAO M-1	3.87	3.66
SAO COSPAR (No 11th)	3.17	- -
SAO 1969	2.80	- -
Köhnlein	11.18	4.01
Rapp	7.68	4.02
NWL 5E-6	12.75	3.50
APL 3.5	12.65	4.53
Kaula	5.43	5.86

*Addition of Gaposchkin & Veis (1967) 12th order terms.

**1 second of arc equals approximately 7 meters.

Table 8
Rms's About Fitted Orbits
GEOS II

ARC 1: April 28 - May 4, 1968
(1098 observations)

Model	Rms (secs of arc)	
	unmodified	modified*
SAO M-1	17.36	3.08
SAO M-1	- -	6.12**
SAO COSPAR	5.54	- -
SAO 1969	2.50	- -
Köhnlein	9.41	3.12
Rapp	11.30	5.48
NWL 5E-6	27.99	8.08
APL 3.5	59.71	5.79
Kaula	16.67	9.32

ARC 2: September 16 - 22, 1968
(1388 observations)

Model	Rms (secs of arc)	
	unmodified	modified*
SAO M-1	12.87	6.11
SAO M-1	- -	5.53**
SAO 1969	2.37	- -
Köhnlein	6.52	- -
Rapp	7.05	- -
NWL 5E-6	21.35	- -
APL 3.5	68.95	- -
Kaula	11.25	- -

*Addition of Yionoulis (1968) + Douglas & Marsh (1969).

**Addition of Yionoulis (1968) only.

For GEOS II, the Yionoulis odd-degree 13th order terms alone greatly reduced the rms of fit (the SAO M-1 result in Table 8 is similar to the results obtained when the other models were modified by Yionoulis values only), but an along track effect of about 150 meters remained. The approximate calculations of Table 3 show that the most important even-degree resonant term for GEOS-II is

(14,13). By analyzing the variation in the along track residuals obtained from the SAO M-1 with Yionoulis 13th order term solution, Douglas and Marsh (1969) produced values of (14,13) which eliminated the effect of the even-degree terms for GEOS-II on the April-May arc. On the September arc, models which contained an insufficient number of resonant terms also were modified by incorporating the Yionoulis and Douglas and Marsh 13th order terms. But when the Douglas and Marsh term plus the Yionoulis coefficients were added to the SAO M-1 model, the rms of fit increased for the September arc, in contrast to the substantial improvement obtained for the April - May arc. This suggests that multi-arc solutions for composite coefficients are required rather than the single arc used by Douglas and Marsh. The arcs should be chosen so that the values of the orientation elements, argument of perigee (ω) and right ascension of the ascending node, (Ω) are as varied as possible.

Preliminary analyses with the SAO COSPAR (1969) model indicated the 11th order coefficients were in error. When the complete COSPAR model was used in obtaining orbital solutions for GEOS-I, the rms's on both arcs were about 10 seconds of arc. Hence, all SAO COSPAR solutions used in this study were obtained without using any 11th order coefficients. The more recent SAO model, the 1969, did not contain any such errors, as shown by the excellent solutions produced by the unmodified version.

3.3 PREDICTION CAPABILITIES

An important application of an accurate geopotential model is its ability to predict the position of a satellite considerably ahead of the last fitted data point. This ability can be measured by computing the root mean squares of observations about a trajectory generated ahead of the fitted orbital arc. Table 9 presents the rms's about the predicted trajectories from the July GEOS-I solutions and the April-May GEOS-II solutions. Prediction results from the GEOS-I and GEOS-II arcs were consistent with the fits in Tables 7 and 8.

The large difference in prediction capability of the GEOS-I solutions and the GEOS-II solutions is immediately obvious. A possible cause of this difference is the fact that GEOS-II is relatively more perturbed by the geopotential than GEOS-I (see Section 1.1); thus a relatively more precise model is necessary for GEOS-II to achieve the same results as GEOS-I. Also the MOTS camera stations (which supplied most of the April - May data) are relatively poorly distributed geographically.

The SAO 1969 model consistently gave better results, both for GEOS-I and II, followed closely by the modified Köhnlein and SAO M-1 models. Prediction results were not obtained for the smaller, unmodified models because the rms's of the fits were so high that reasonable predictions would be unlikely.

3.4 SATELLITE POSITION COMPARISONS

Another method of comparing orbits determined with different gravity models consists of taking computed satellite positions determined with different models and differencing them. The

satellite position differences are resolved into radial, cross track and along track components. This method is very useful in spotting differences in orbits due to resonance.

In this study, all GEOS-I orbits were compared against the orbit determined by the SAO M-1 with Gaposchkin and Veis 12th order terms. The GEOS-II orbits were compared with the orbit determined by the SAO M-1 with the Yionoulis and Douglas and Marsh 13th order terms. This does not imply that these models are always "best" in every sense. We chose the M1 model as the basis for comparison because it is so widely used.

Table 9
Fits About Predicted Trajectories GEOS-I and GEOS-II

GEOS-I, ARC 1

Definitive Period: July 11-16, 1966 (1766 Obs.)

Prediction Period: July 17-22, 1966 (1858 Obs.)

Model	Rms (secs of arc)	
	unmodified	modified*
SAO M-1	- - -	5.88
SAO 1969	4.75	- - -
Köhnlein	- - -	5.25
Rapp	- - -	23.68
NWL 5E-6	- - -	7.08
APL 3.5	- - -	20.57
Kaula	6.58	7.43

GEOS-II, ARC 1

Definitive Period: April 28-May 4, 1968 (1098 Obs.)

Prediction Period: May 5-9, 1968 (622 Obs.)

Model	Rms (secs of arc)	
	unmodified	modified**
SAO M-1	- - -	12.17
SAO 1969	13.04	-----
Köhnlein	28.16	11.20
Rapp	33.51	13.89
NWL 5E-6	- - -	31.87
APL 3.5	- - -	22.63
Kaula	78.09	17.09

*Addition of Gaposchkin & Veis (1967)

**Addition of Yionoulis (1968) and Douglas & Marsh (1969)

Table 10 summarizes the results of the gravity model comparisons over the fitted arcs. As expected, the along track differences were more outstanding than the radial and cross track differences (see Section 1.2), even though the radial and cross track differences were larger for GEOS-II orbits than for GEOS I. This is probably due to the less widely distributed tracking stations for the GEOS II arc and the fact that GEOS-II is slightly more perturbed by the geopotential than is GEOS-I. The one exception to this is the SAO M-1 modified versus the NWL 5E-6 modified orbits on the GEOS-II arc, where the cross track differences are frequently as great as 200 meters. The along track differences are only occasionally that large. Table 10 reveals something very important about orbit determination accuracy. We see, for example, that for GEOS-II, the orbit obtained with the modified SAO M1 model differs along-track from the orbit obtained with the SAO 1969 model by about 64 m (rms). Although the SAO 1969 model gives a better fit, the modified M1 model gives better predictions. Thus we cannot directly say which is "best". Figure 1 may shed some light. It shows the apparent timing errors obtained for passes of range data recorded from the Rosman, N. C., S-Band Radar tracking site based upon the April - May GEOS-II optical arc using the SAO 1969 and Modified M1 models. The range data were not used in the determination of the optical reference orbit. These "timing errors" are of course due to unmodeled orbit variations, and not to system or hardware errors and in the case of the SAO 1969 model are obviously due mainly to inadequately modeled resonance in the amount of about 40 m along-track.

The results in Table 10 are consistent with previous tables of fitted and predicted orbits. Considering results for both GEOS I and II, they suggest that for 5 - 6 day arcs, we cannot be certain of satellite position to perhaps 50-100 meters (with published gravity models) even during the period of observation. The need for improved gravity models is unquestionable.

Figures 2 (a - c) present the first, third and sixth day plots of along track differences presented in Table 10 for the three best GEOS-I orbits: the modified Köhnelein, the modified NWL 5E-6 and the SAO 1969. Figures 3 (a - c) present the along track differences for the same models for the GEOS-II April - May arc. Again note that the modified M-1 and SAO-1969 orbits differ by over 100 meters fairly regularly for both GEOS-I and GEOS-II comparisons.

Figures 4 (a-f) contain a plot comparing the GEOS-I unmodified Köhnelein and the modified Köhnelein along track differences for the period July 11 - 16, 1966. The six-day period of the differences of the unmodified model clearly indicate that they are due to inadequate 12th order coefficients. Addition of the Gaposchkin and Veis 12th order terms greatly reduced these differences. On the basis of the rms's of fits (Table 7) we can say that the Köhnelein orbit was significantly improved. Similar reductions in along track differences were seen in nearly every such comparison of modified and unmodified models. Of course those gravity models that contain no GEOS resonant terms give very poor results.

Table 10
Rms's of Position Differences
GEOS I (July 11 - 16, 1966) and GEOS II (April 28 - May 4, 1968)

SAO M-1 (modified*) vs.	Position (meters)							
	GEOS I				GEOS II			
	Radial	Cross Track	Along Track	Total	Radial	Cross Track	Along Track	Total
SAO M-1 (unmodified)	30.0	17.1	286.4	288.5	22.3	10.3	232.2	234.2
SAO COSPAR (no. 11th)	8.9	12.8	29.5	33.4	24.7	18.2	92.8	97.8
SAO 1969	8.3	13.5	26.9	31.2	16.6	19.3	59.2	64.4
Köhnlein	16.4	16.1	213.0	214.2	20.1	16.9	129.1	131.7
Köhnlein (modified)	9.2	10.9	28.0	31.4	11.8	15.1	41.9	46.1
Rapp	48.3	29.5	129.4	141.2	38.8	37.7	157.6	166.6
Rapp (modified)	46.4	33.2	99.9	115.0	36.2	39.1	84.8	100.1
APL 3.5	46.1	46.8	175.7	187.1	71.8	55.2	674.3	680.4
APL 3.5 (modified)	42.5	41.6	90.1	107.9	34.7	45.4	88.1	105.2
NWL 5E-6	16.3	16.6	204.0	205.3	46.4	80.9	374.6	386.1
NWL 5E-6 (modified)	16.7	12.9	49.1	53.4	26.3	80.0	82.9	118.2
Kaula	32.5	42.2	114.1	125.9	47.6	43.5	232.8	241.4
Kaula (modified)	32.1	42.5	110.2	122.4	48.7	42.0	140.5	154.6

*Gaposchkin & Veis (1967) 12th order terms for GEOS-1 and Yionoulis (1968)
and Douglas & Marsh (1968) 13th order terms for GEOS-11.

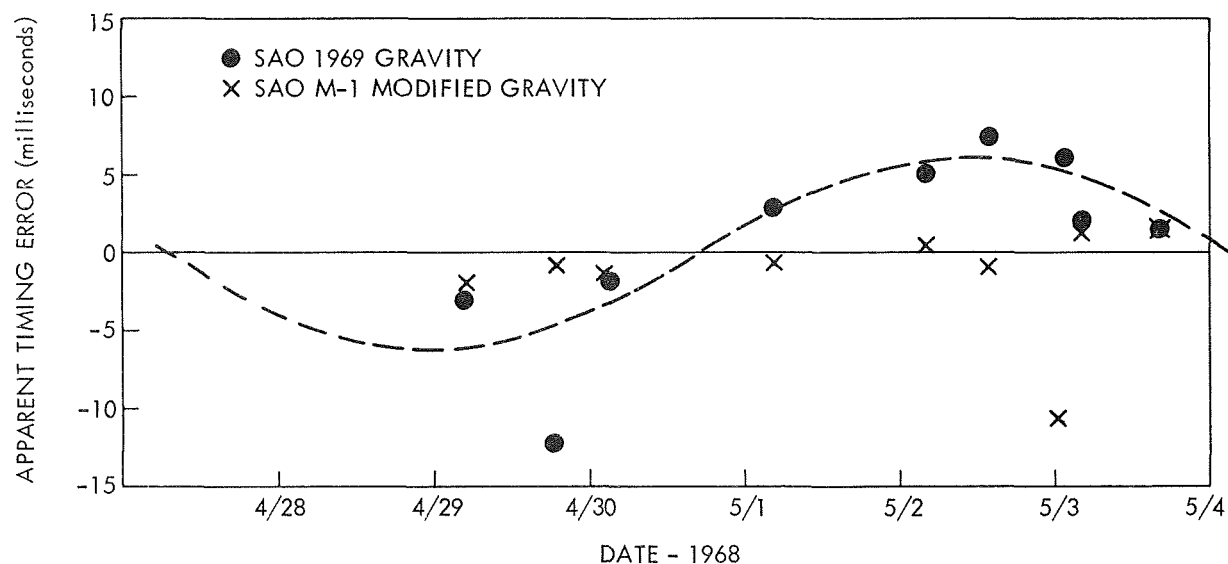


Figure 1. Apparent Timing Errors in Rosman Range Data Due to Unmodeled Orbit Variations Based Upon 6 Day GEOS-II Optical Reference Orbits

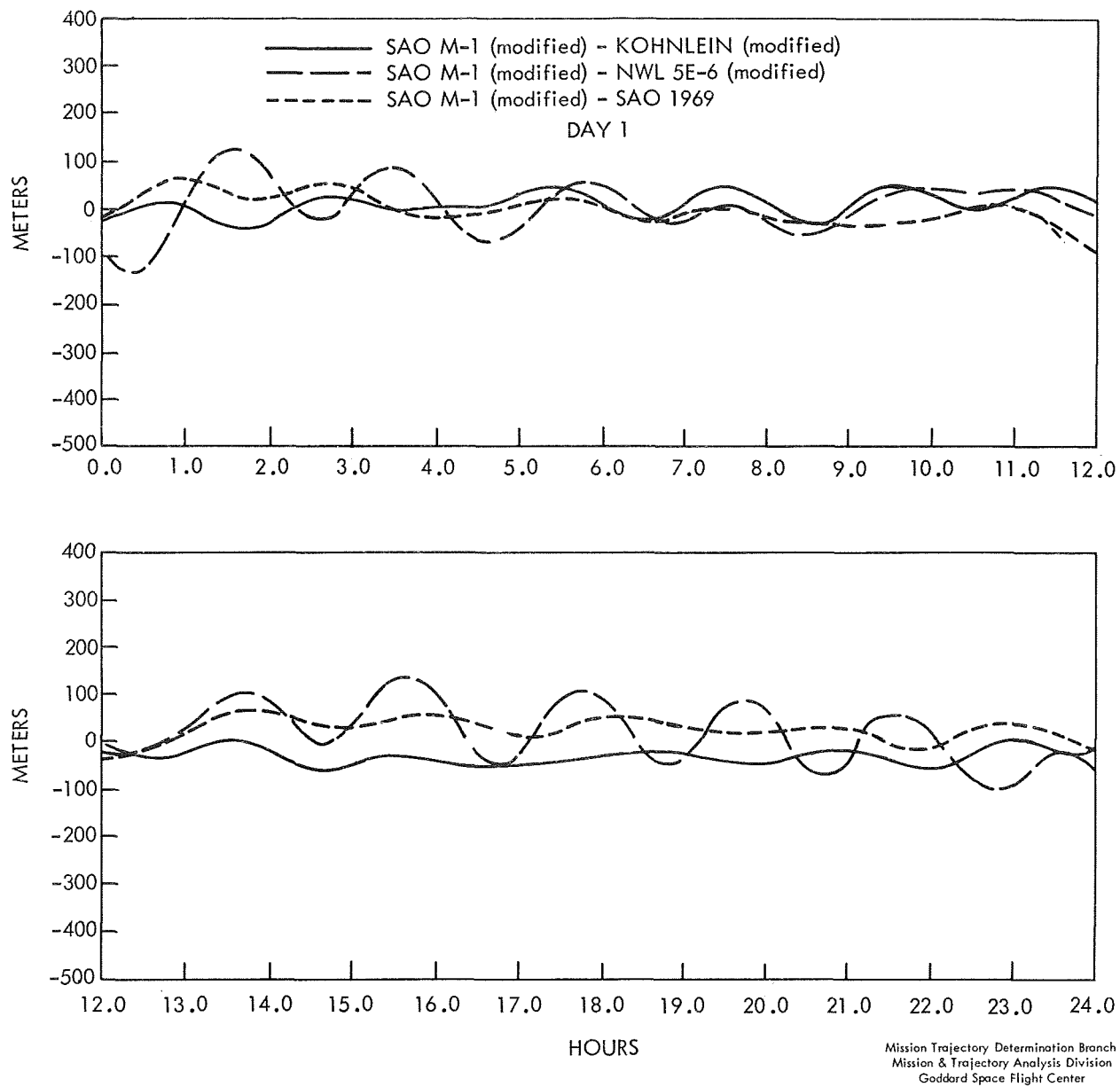


Figure 2(a). Along Track Position Differences – GEOS-I July 11-16, 1966

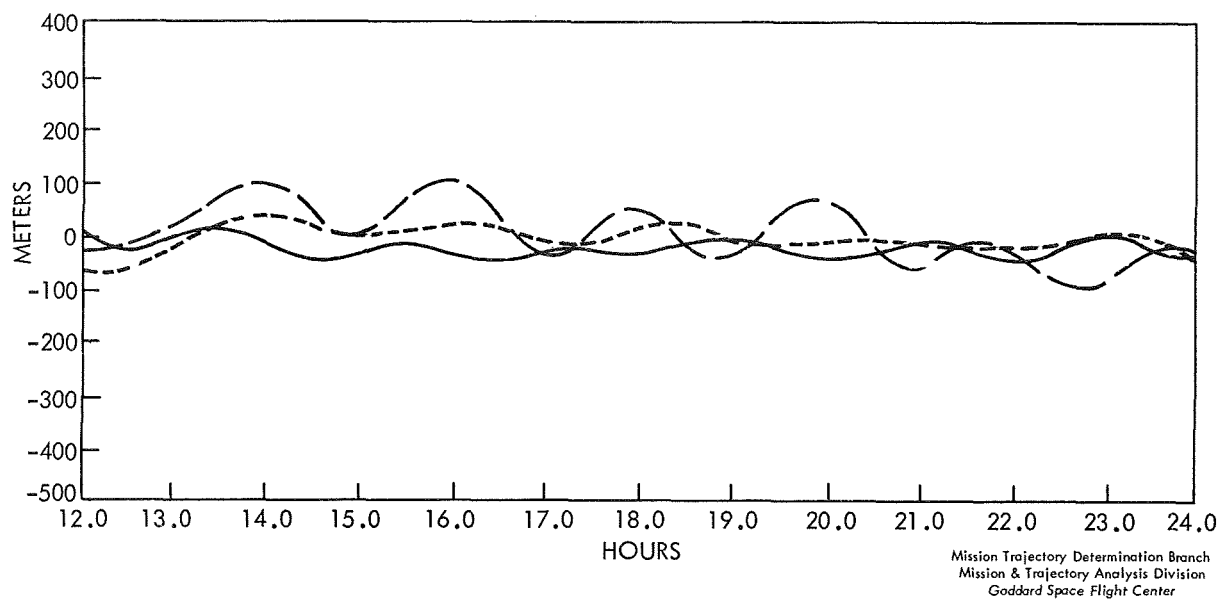
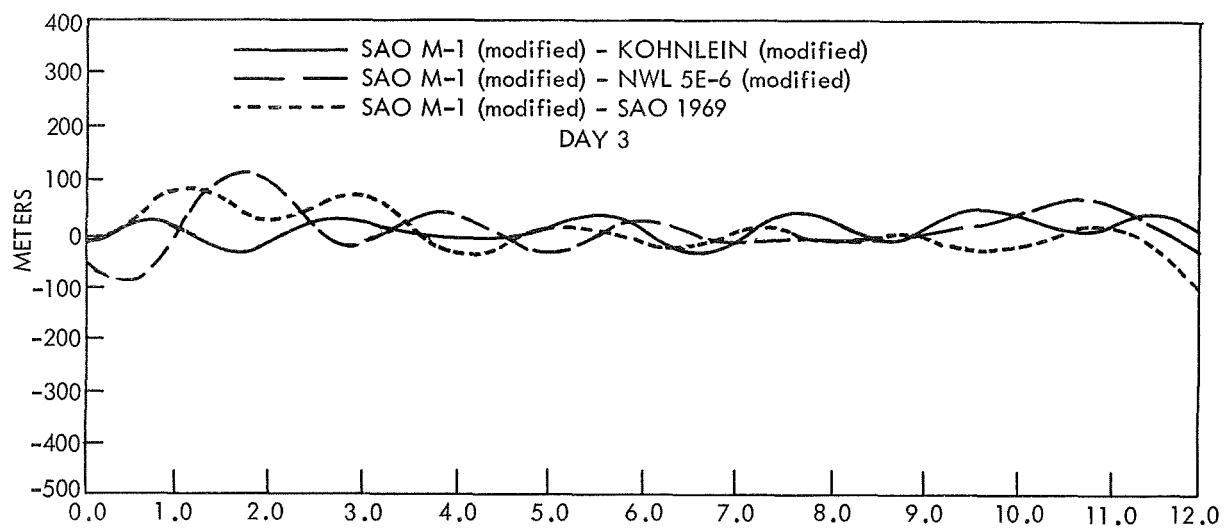


Figure 2(b). Along Track Position Differences - GEOS-1 July 11-16, 1966

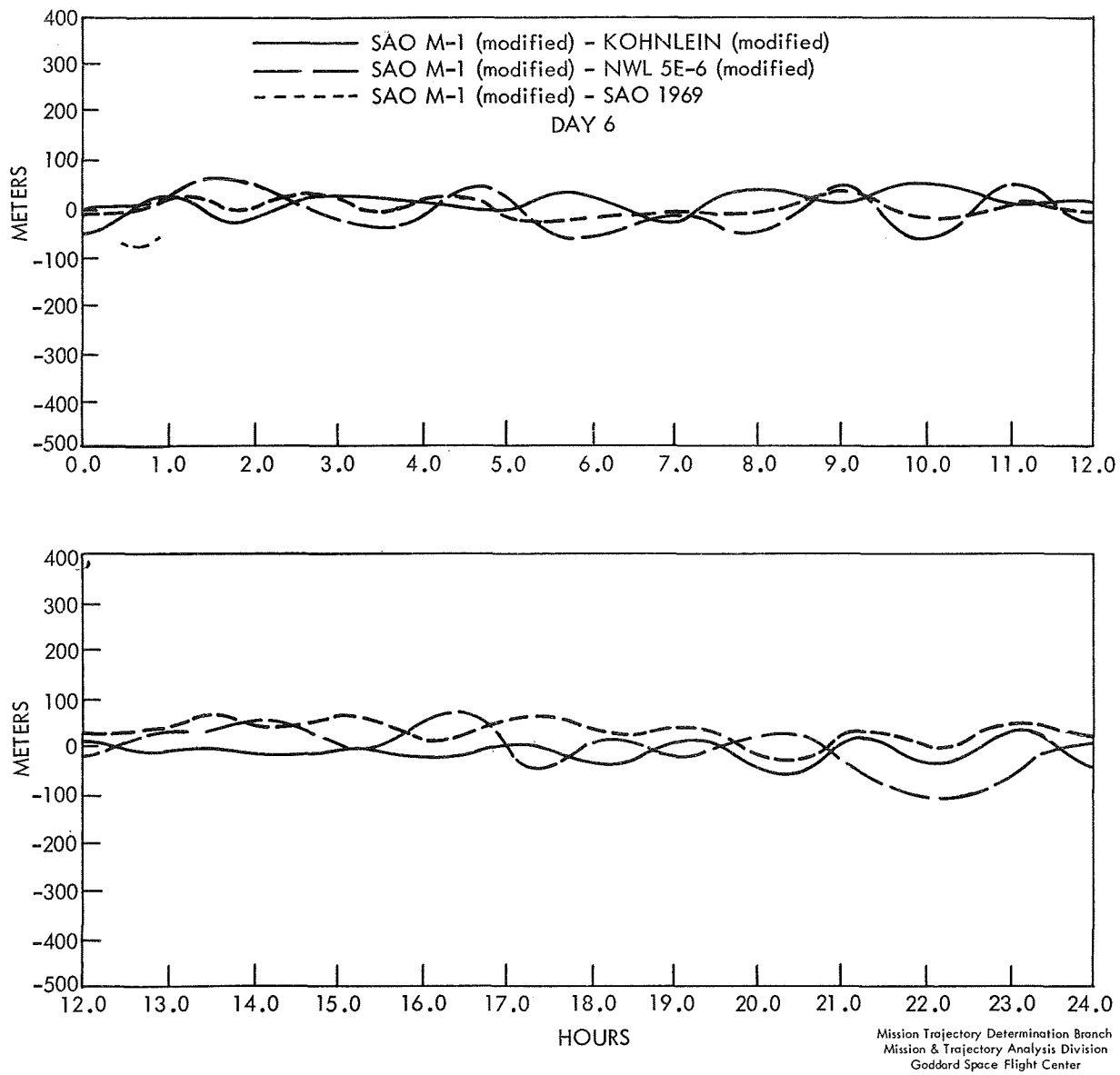


Figure 2(c). Along Track Position Differences – GEOS-I July 11-16, 1966

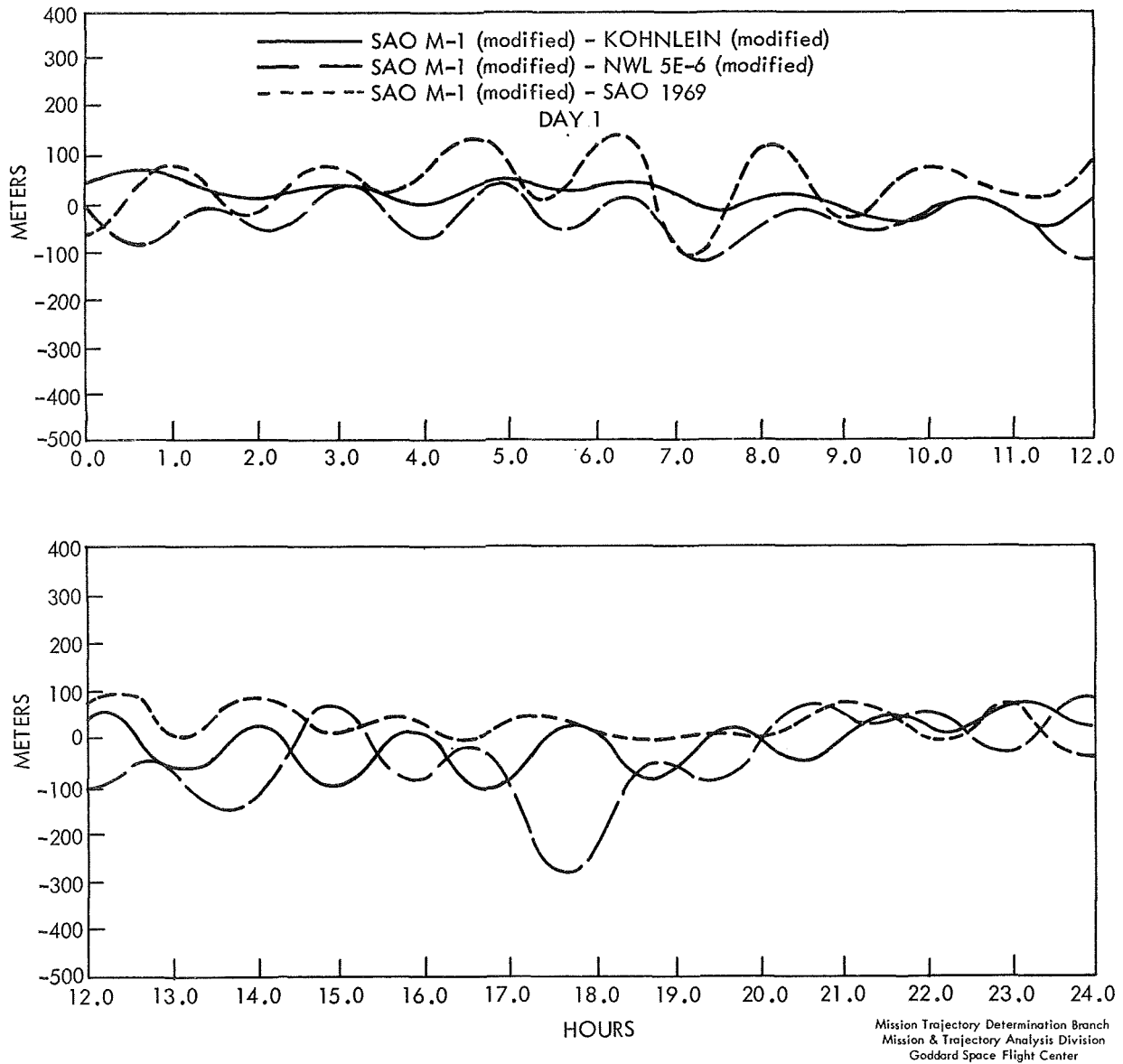


Figure 3(a). Along Track Position Differences - GEOS-II April 28-May 4, 1968

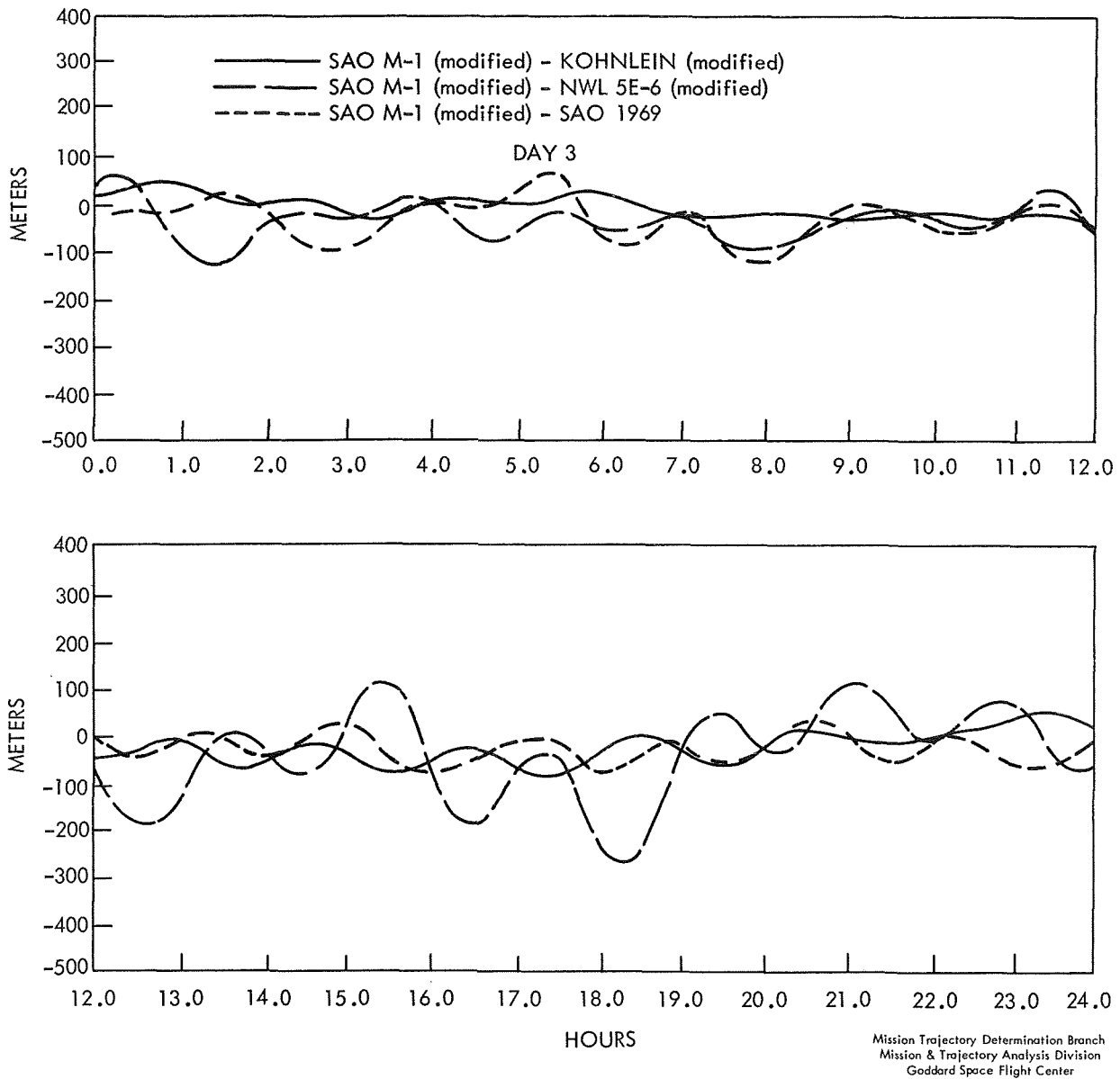


Figure 3(b). Along Track Position Differences – GEOS-II April 28-May 4, 1968

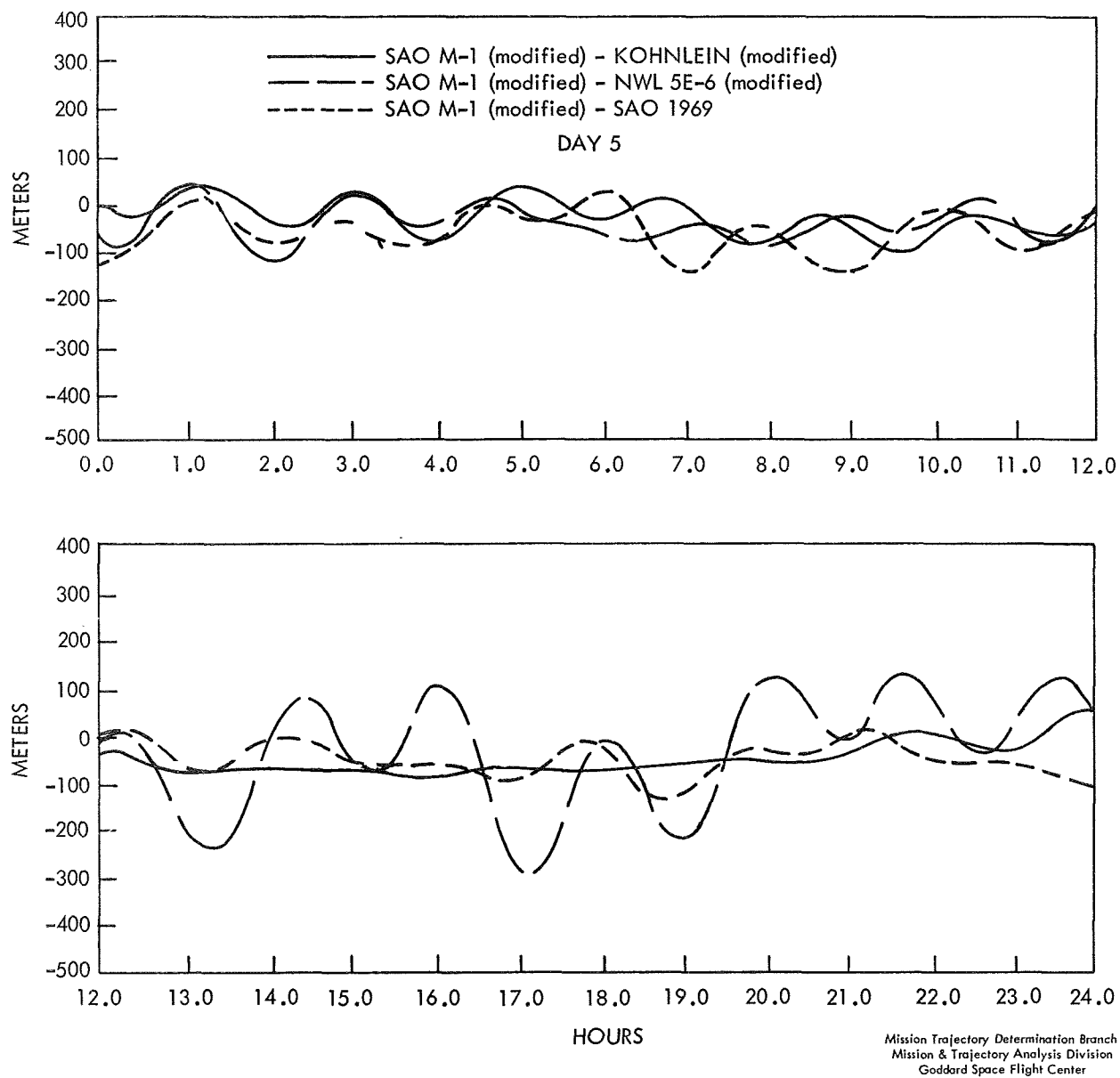


Figure 3(c). Along Track Position Differences – GEOS-II April 28-May 4, 1968

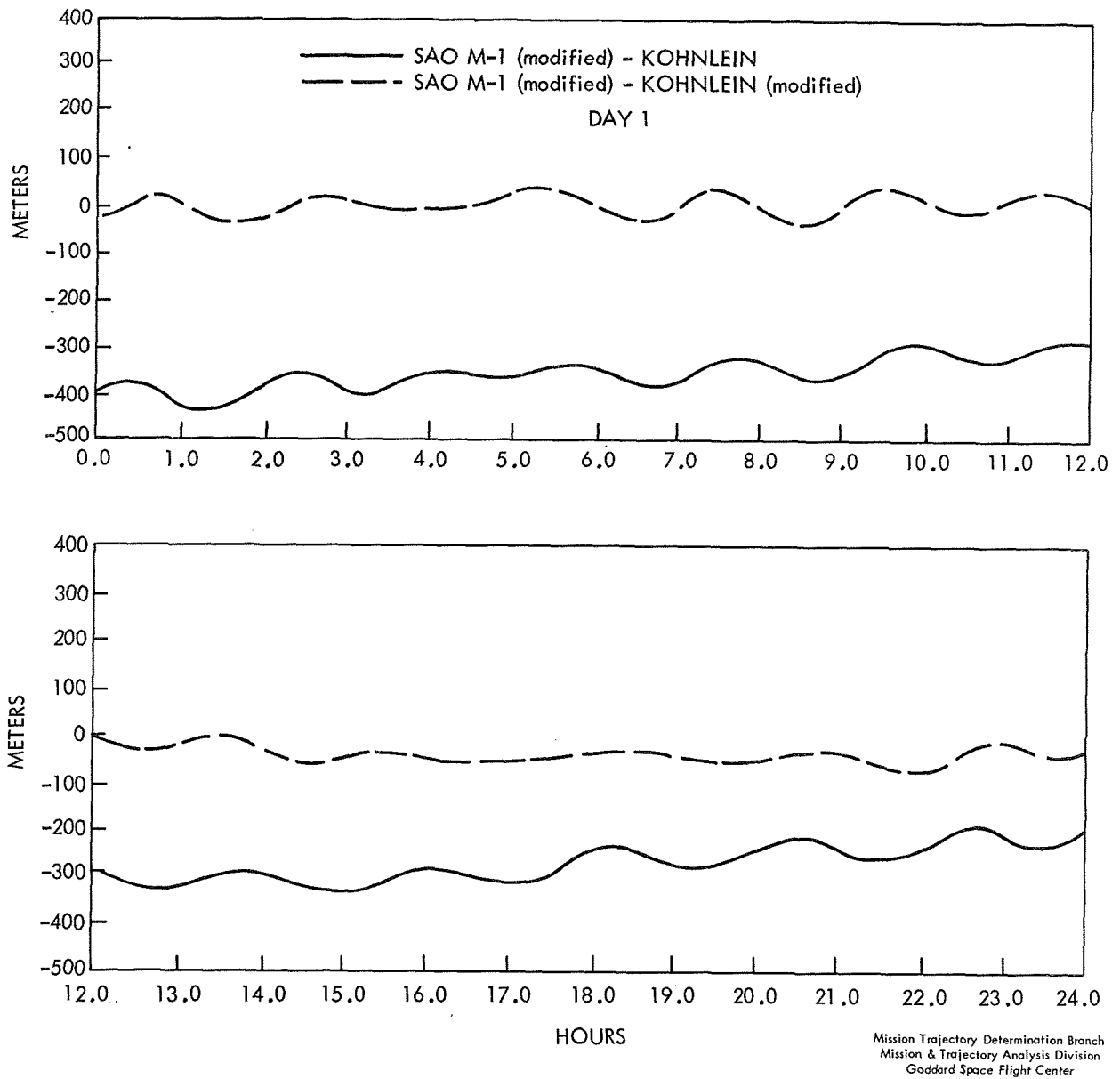


Figure 4(a). Along Track Position Differences - GEOS-1 July 11-16, 1966

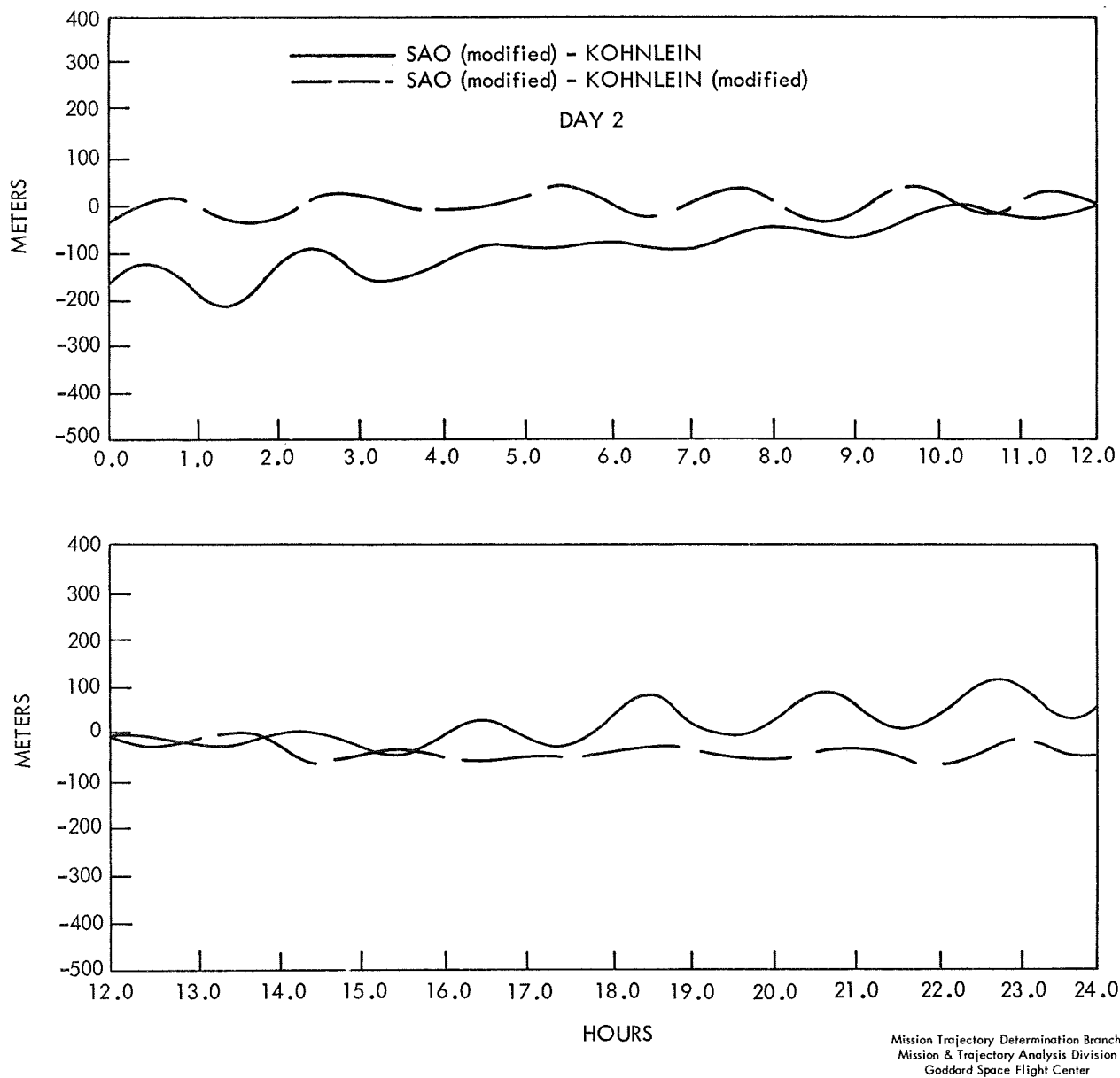


Figure 4(b). Along Track Position Differences – GEOS-I July 11-16, 1966

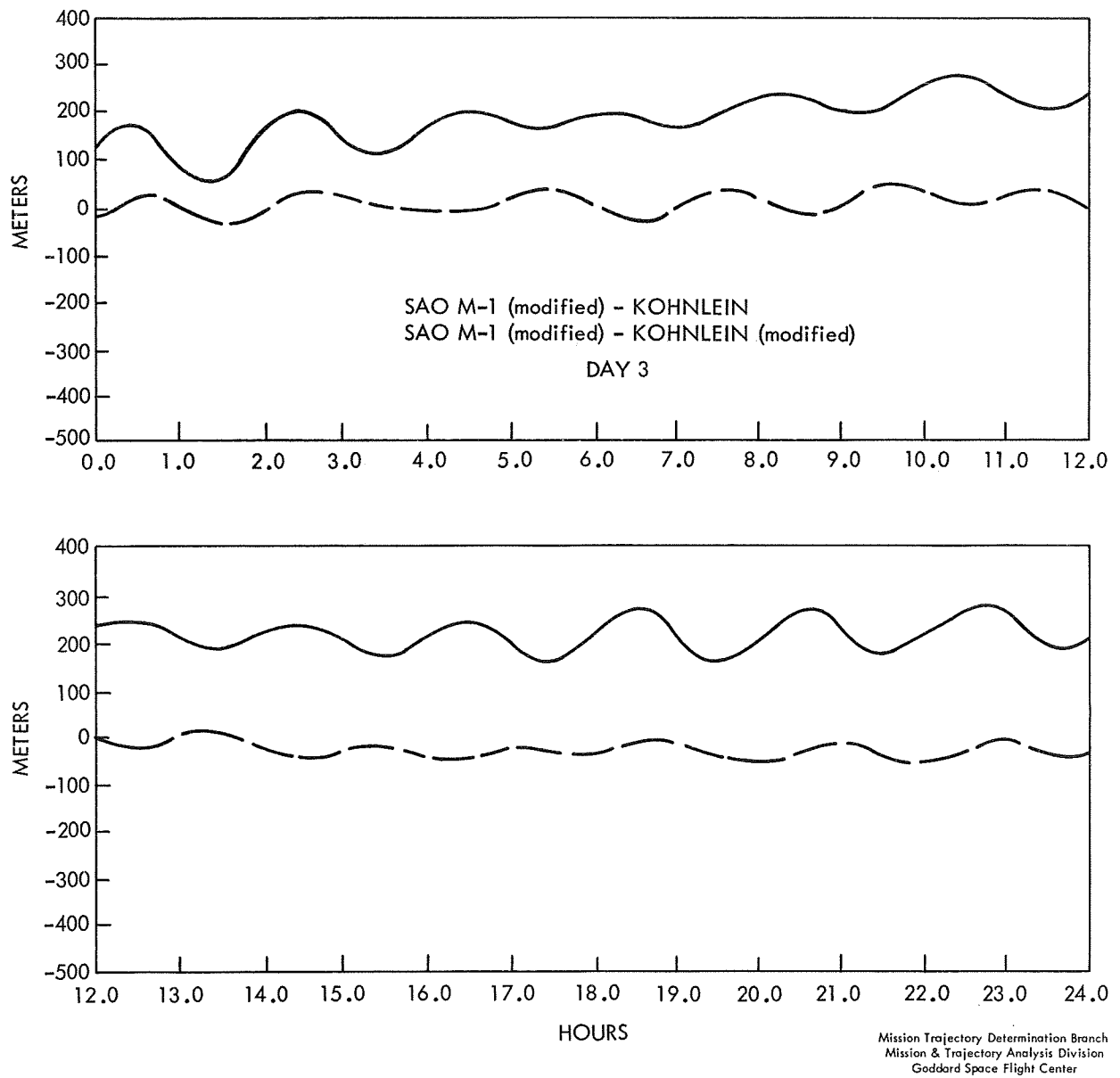
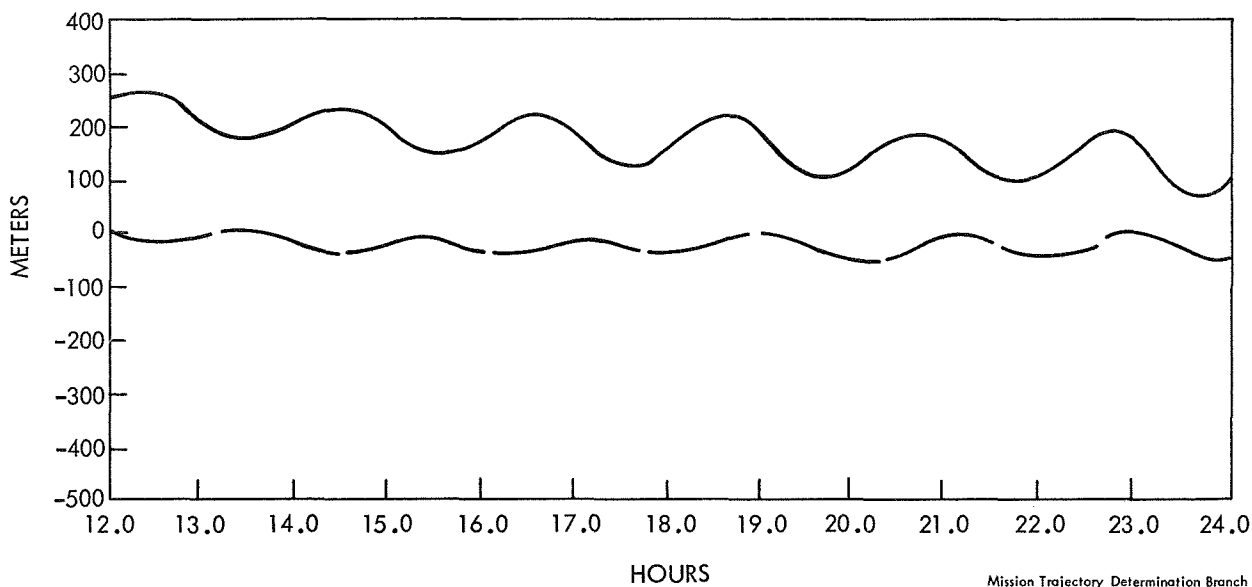
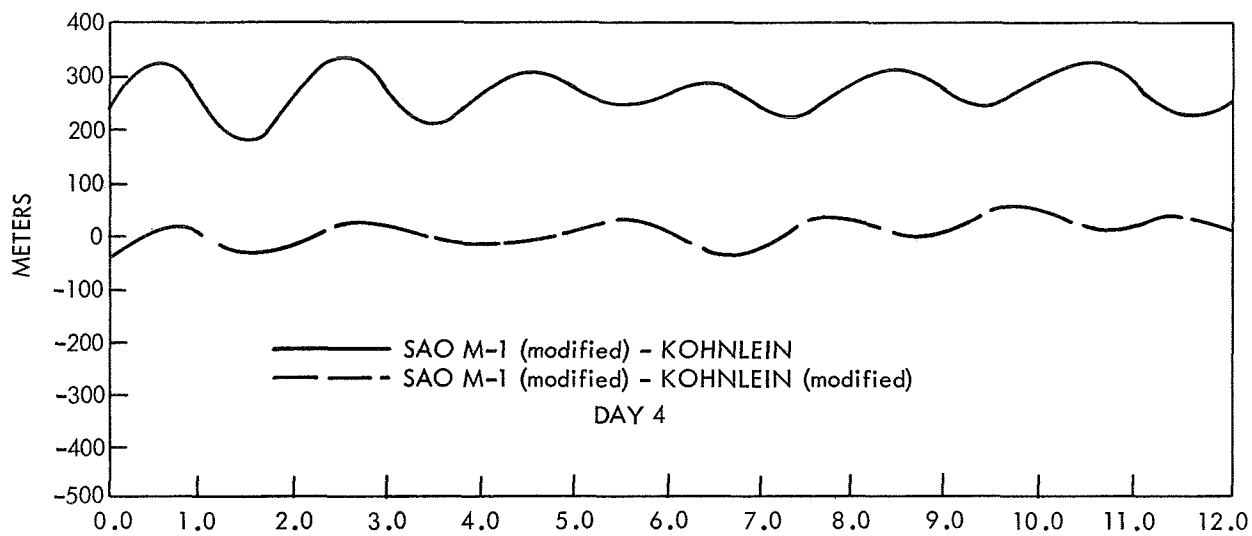


Figure 4(c). Along Track Position Differences – GEOS-I July 11-16, 1966



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Figure 4(d). Along Track Position Differences – GEOS-I July 11-16, 1966

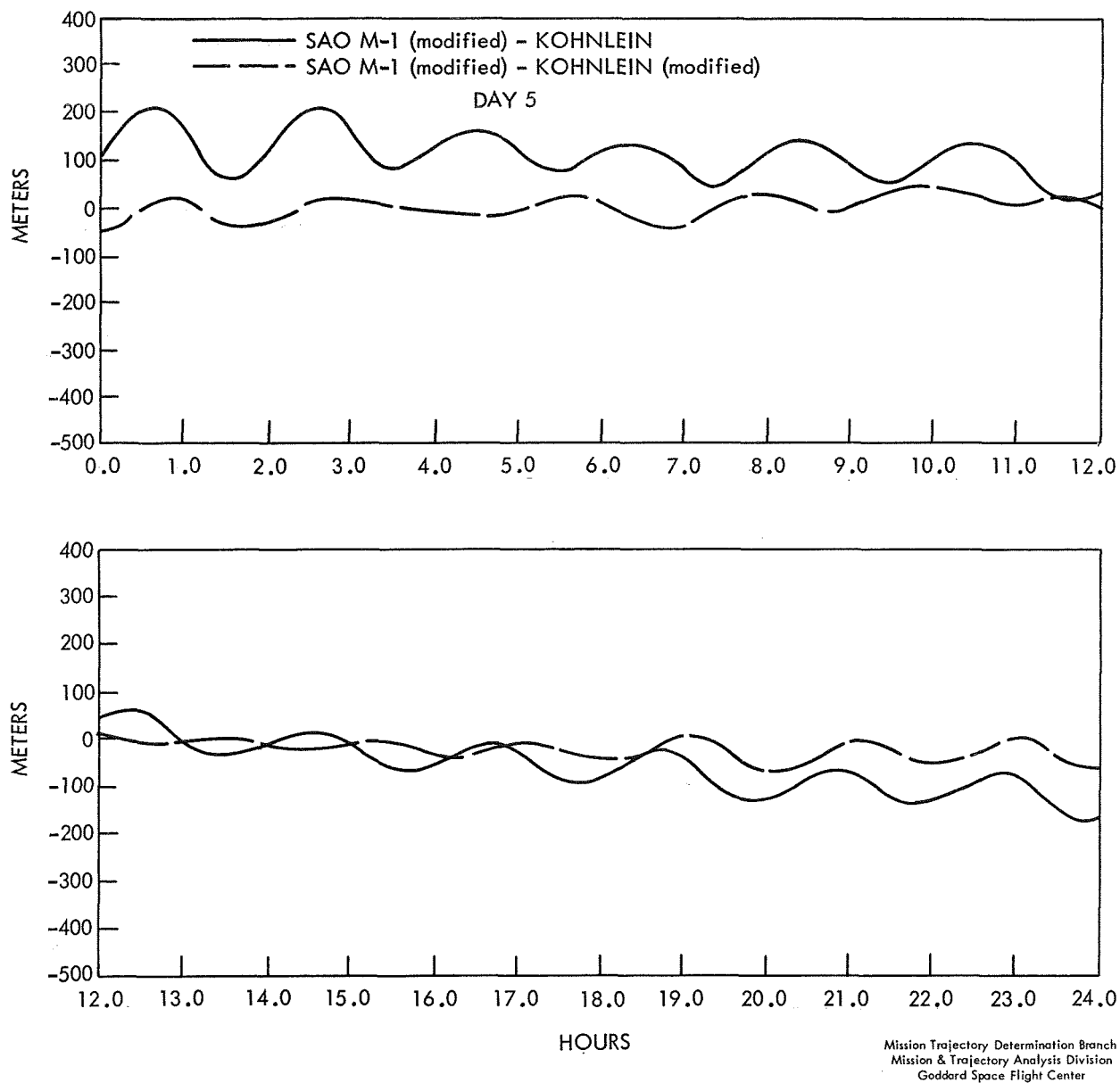
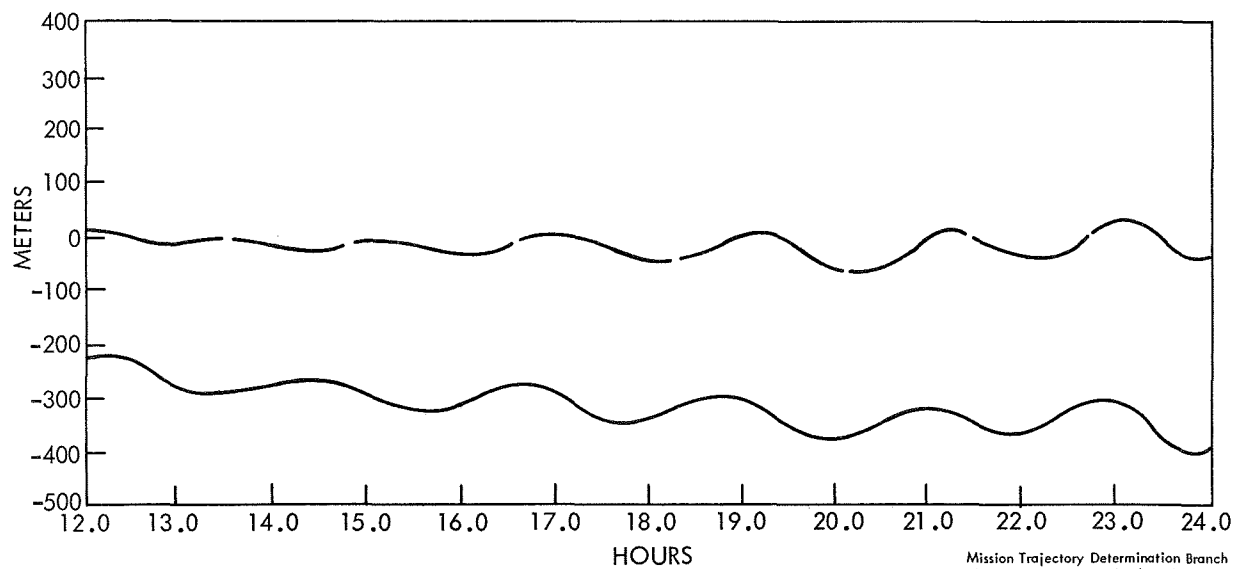
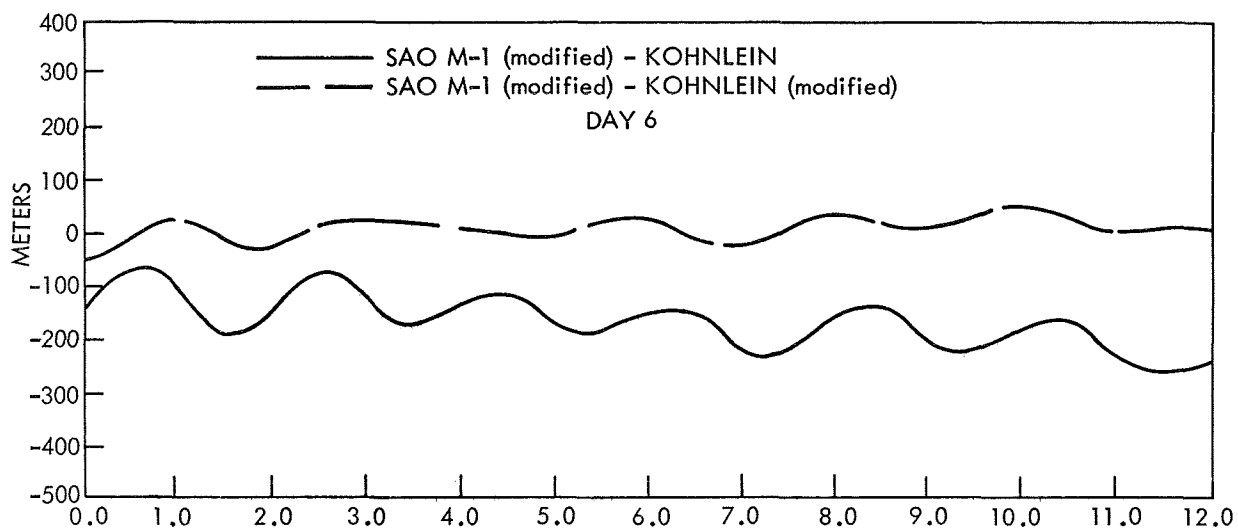


Figure 4(e). Along Track Position Differences - GEOS-1 July 11-16, 1966



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Figure 4(f). Along Track Position Differences - GEOS-1 July 11-16, 1966

SECTION 4

CONCLUSIONS

The most obvious (and not so surprising) result of this study is that resonance must be accurately modeled for precision orbit determination. Both GEOS-I and GEOS-II orbits were greatly improved by using improved resonant coefficients. Also, the resonance effect can be modeled for a particular orbital arc by one or two "lumped" coefficients; however, the generality of such a solution is in question. The 1966 SAO M-1 12th order coefficients gave good results over the December-January arc but a high rms of fit in the July arc. The Douglas and Marsh values for (14,13) greatly improved the April-May, 1968, GEOS-II orbits but seemed to worsen the September, 1968, SAO M-1 and 1969 orbits. The very good results obtained when using the SAO 1969 model are almost certainly due to the richly varied resonant orbits used in the derivation of this model.

Based on the results presented in this study, the SAO 1969 geopotential model is a significant improvement over the 1966 SAO M-1 model, and is the most accurate model published to date. Yet improvement is still possible; note the unmodeled resonance effect in Figure 1 and the superior orbital prediction given by the modified M1 model for GEOS-II.

The good results obtained with the modified Kohnlein and Rapp and the SAO 1969 also indicate that gravimetric data may be used to good advantage in perfecting geopotential models for satellite orbit determination. As noted by Douglas and Marsh (Reference 12), the results for the Kohnlein and Rapp models demonstrate that gravimetric data has provided estimates of 12th and 13th order coefficients that remove much of the resonance effect for the GEOS satellites.

Finally, we are forced to conclude that even with the best available gravity models and observing equipment, satellite position along-track is uncertain at various points on the orbit by 50-100 meters for 5-6 day arcs. A degradation will be observed for longer arcs.

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